

Precalculus

5-01 Fundamental Trigonometric Identities Part A

Uses for identities

- _____ trig functions
- _____ trig expressions
- Develop more _____
- _____ trig equations

Reciprocal Identities

$$\sin u = \frac{1}{\csc u}$$

$$\csc u = \frac{1}{\sin u}$$

$$\cos u = \frac{1}{\sec u}$$

$$\sec u = \frac{1}{\cos u}$$

$$\tan u = \frac{1}{\cot u}$$

$$\cot u = \frac{1}{\tan u}$$

Quotient Identities

$$\tan u = \frac{\sin u}{\cos u}$$

$$\cot u = \frac{\cos u}{\sin u}$$

Pythagorean Identities

$$\sin^2 u + \cos^2 u = 1$$

$$\tan^2 u + 1 = \sec^2 u$$

$$1 + \cot^2 u = \csc^2 u$$

Even/Odd Identities

$$\cos(-u) = \cos u$$

$$\sec(-u) = \sec u$$

$$\sin(-u) = -\sin u$$

$$\tan(-u) = -\tan u$$

$$\csc(-u) = -\csc u$$

$$\cot(-u) = -\cot u$$

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - u\right) = \cos u$$

$$\cos\left(\frac{\pi}{2} - u\right) = \sin u$$

$$\tan\left(\frac{\pi}{2} - u\right) = \cot u$$

$$\cot\left(\frac{\pi}{2} - u\right) = \tan u$$

$$\sec\left(\frac{\pi}{2} - u\right) = \csc u$$

$$\csc\left(\frac{\pi}{2} - u\right) = \sec u$$

If $\sin \theta = -1$ and $\cot \theta = 0$, evaluate $\cos \theta$

Evaluate $\tan \theta$

Simplify $\frac{\sec^2 x - 1}{\sin^2 x}$

Simplify $\sin \varphi (\csc \varphi - \sin \varphi)$

Simplify $\frac{1 - \sin^2 x}{\csc^2 x - 1}$

Simplify $\cos\left(\frac{\pi}{2} - x\right) (\sec x)$

Precalculus

5-02 Fundamental Trigonometric Identities Part B

Factor and simplify $\sin^4 x - \cos^4 x$

Multiply and simplify $(2 \csc x + 2)(2 \csc x - 2)$

Simplify $\frac{\cos x}{1 + \sin x} + \frac{1 + \sin x}{\cos x}$

Rewrite not as a fraction: $\frac{3}{\sec x - \tan x}$

Use trig substitution: $\sqrt{x^2 - 9}$ with $x = 3 \sec \theta$

Precalculus

5-03 Verify Trigonometric Identities

Verifying Trig Identities

- Show that trig identities are _____ by _____ one side into the other side

Guidelines

- Work with _____ at a time. Start with the more _____ side.
- Try _____, add _____, square a _____, etc.
- Use fundamental _____
- If the above doesn't work, try rewriting in _____ and _____
- Try _____!

Verify $(1 + \sin \alpha)(1 - \sin \alpha) = \cos^2 \alpha$

Verify $\sin^2 \alpha - \sin^4 \alpha = \cos^2 \alpha - \cos^4 \alpha$

Verify $\frac{\cot^2 t}{\csc t} = \csc t - \sin t$

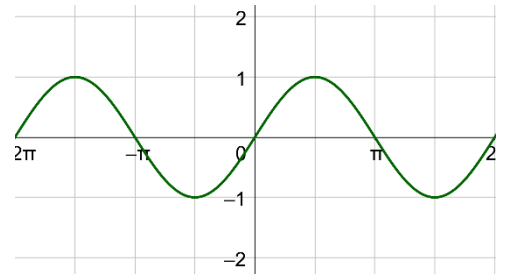
Verify $\frac{1}{\sec x \tan x} = \csc x - \sin x$

Verify $\frac{\cos \theta \cot \theta}{1 - \sin \theta} - 1 = \csc \theta$

Precalculus

5-04 Solve Trigonometric Equations

- Main goal – Isolate a _____ expression
 - Try _____ to simplify
 - Try solving by _____



Number of solutions

- $\sin x = 0$
- Infinite solutions so describe
- _____

Solve $\sin x - \sqrt{2} = -\sin x$

Solve $4 \sin^2 x - 3 = 0$

Solve $\sin^2 x = 2 \sin x$

$$\text{Solve } 3 \sec^2 x - 2 \tan^2 x - 4 = 0$$

$$\text{Solve in the interval } [0, 2\pi): \sin x + 1 = \cos x$$

$$\text{Solve on the interval } [0, 2\pi): \sin 2x = \frac{\sqrt{3}}{2}$$

$$\text{Solve } 4 \tan^2 x + 5 \tan x = 6$$

Precalculus

5-05 Sum and Difference Formulas

Sum and Difference Formulas

$$\sin(u + v) = \sin u \cos v + \cos u \sin v$$

$$\sin(u - v) = \sin u \cos v - \cos u \sin v$$

$$\cos(u + v) = \cos u \cos v - \sin u \sin v$$

$$\cos(u - v) = \cos u \cos v + \sin u \sin v$$

$$\tan(u + v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$$

$$\tan(u - v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$$

Use a sum or difference formula to find the exact value of $\tan 255^\circ$

Find the exact value of $\cos 95^\circ \cos 35^\circ + \sin 95^\circ \sin 35^\circ$

Derive a reduction formula for $\sin\left(t + \frac{\pi}{2}\right)$

Find all solutions in $[0, 2\pi)$: $\cos\left(x - \frac{\pi}{3}\right) + \cos\left(x + \frac{\pi}{3}\right) = 1$

Precalculus

5-06 Multiple Angle Formulas

Double-Angle Formulas

- $\sin 2u = 2 \sin u \cos u$
- $\cos 2u = \cos^2 u - \sin^2 u$
 $= 2 \cos^2 u - 1$
 $= 1 - 2 \sin^2 u$
- $\tan 2u = \frac{2 \tan u}{1 - \tan^2 u}$

If $\sin u = \frac{3}{5}$ and $0 < u < \frac{\pi}{2}$,

Find $\sin 2u$

$\cos 2u$

$\tan 2u$

Derive a triple angle formula for $\cos 3x$

Power-Reducing Formulas

- $\sin^2 u = \frac{1 - \cos 2u}{2}$
- $\cos^2 u = \frac{1 + \cos 2u}{2}$
- $\tan^2 u = \frac{1 - \cos 2u}{1 + \cos 2u}$

Rewrite $\cos^4 x$ as a sum of 1st powers of cosines.

Half-Angle Formulas

- $\sin \frac{u}{2} = \pm \sqrt{\frac{1 - \cos u}{2}}$
- $\cos \frac{u}{2} = \pm \sqrt{\frac{1 + \cos u}{2}}$
- $\tan \frac{u}{2} = \frac{1 - \cos u}{\sin u}$
 $= \frac{\sin u}{1 + \cos u}$

Find the exact value of $\cos 105^\circ$

Precalculus

5-07 Product-to-Sum Formulas

Product-to-Sum Formulas

- $\sin u \sin v = \frac{1}{2}(\cos(u - v) - \cos(u + v))$
- $\cos u \cos v = \frac{1}{2}(\cos(u - v) + \cos(u + v))$
- $\sin u \cos v = \frac{1}{2}(\sin(u + v) + \sin(u - v))$
- $\cos u \sin v = \frac{1}{2}(\sin(u + v) - \sin(u - v))$

Rewrite $\sin 5\theta \cos 3\theta$ as a sum or difference.

Sum-to-Product Formulas

- $\sin u + \sin v = 2 \sin\left(\frac{u+v}{2}\right) \cos\left(\frac{u-v}{2}\right)$
- $\sin u - \sin v = 2 \cos\left(\frac{u+v}{2}\right) \sin\left(\frac{u-v}{2}\right)$
- $\cos u + \cos v = 2 \cos\left(\frac{u+v}{2}\right) \cos\left(\frac{u-v}{2}\right)$
- $\cos u - \cos v = -2 \sin\left(\frac{u+v}{2}\right) \sin\left(\frac{u-v}{2}\right)$

Find the exact value of $\sin 195^\circ + \sin 105^\circ$

Solve on the interval $[0, 2\pi)$: $\sin 4x - \sin 2x = 0$

Verify $\frac{\sin 6x + \sin 4x}{\cos 6x + \cos 4x} = \tan 5x$