Mr. Wright's Math Extravaganza

Precalculus
Analytic Trigonometry

Level 2.0: 70% on test, Level 3.0: 80% on test, Level 4.0: level 3.0 and success on applications

Score	I Can Statements
4.0	☐ I can demonstrate in-depth inferences and applications that go beyond what was taught.
3.5	In addition to score 3.0 performance, partial success at score 4.0 content.
3.0	☐ I can verify trigonometric identities. ☐ I can solve trigonometric equations.
2.5	No major errors or omissions regarding score 2.0 content, and partial success at score 3.0 content.
2.0	☐ I can use identities to evaluate trigonometric functions ☐ I can use identities to simplify trigonometric expressions. ☐ I can verify trigonometric identities graphically. ☐ I can use the sum and difference formulas. ☐ I can use the multiple angle formulas. ☐ I can use the product-to-sum formulas.
1.5	Partial success at score 2.0 content, and major errors or omissions regarding score 3.0 content.
1.0	With help, partial success at score 2.0 content and score 3.0 content.
0.5	With help, partial success at score 2.0 content but not at score 3.0 content.
0.0	Even with help, no success.

Richard Wright

Precalculus

5-01 Fundamental Trigonometric Identities Part A

Uses for identities

- _____trig functions
- _____trig expressions
- Develop more _____
- _____trig equations

Reciprocal Identities

$\sin u = \frac{1}{\csc u}$ $\quad \csc u = \frac{1}{\sin u}$

$\cos u = \frac{1}{\sec u}$ $\quad \sec u = \frac{1}{\cos u}$

$\tan u = \frac{1}{\cot u}$ $\quad \cot u = \frac{1}{\tan u}$

Quotient Identities

$\tan u = \frac{\sin u}{\cos u}$ $\quad \cot u = \frac{\cos u}{\sin u}$

Pythagorean Identities

$\sin^2 u + \cos^2 u = 1$ $\quad \tan^2 u + 1 = \sec^2 u$

$1 + \cot^2 u = \csc^2 u$

Even/Odd Identities

$\cos(-u) = \cos u$ $\quad \sec(-u) = \sec u$

$\sin(-u) = -\sin u$ $\quad \tan(-u) = -\tan u$

$\csc(-u) = -\csc u$ $\quad \cot(-u) = -\cot u$

Cofunction Identities

$\sin\left(\frac{\pi}{2} - u\right) = \cos u$ $\quad \cos\left(\frac{\pi}{2} - u\right) = \sin u$

$\tan\left(\frac{\pi}{2} - u\right) = \cot u$ $\quad \cot\left(\frac{\pi}{2} - u\right) = \tan u$

$\sec\left(\frac{\pi}{2} - u\right) = \csc u$ $\quad \csc\left(\frac{\pi}{2} - u\right) = \sec u$

If $\sin\theta = -1$ and $\cot\theta = 0$, evaluate $\cos\theta$

Evaluate $\tan\theta$

Simplify $\frac{\sec^2 x - 1}{\sin^2 x}$

Simplify $\sin\varphi\,(\csc\varphi - \sin\varphi)$

Simplify $\frac{1-\sin^2 x}{\csc^2 x-1}$

Simplify $\cos\left(\frac{\pi}{2} - x\right)(\sec x)$

Precalculus

5-02 Fundamental Trigonometric Identities Part B

Factor and simplify $\sin^4 x - \cos^4 x$

Multiply and simplify $(2\csc x + 2)(2\csc x - 2)$

Simplify $\frac{\cos x}{1+\sin x} + \frac{1+\sin x}{\cos x}$

Rewrite not as a fraction: $\frac{3}{\sec x - \tan x}$

Use trig substitution: $\sqrt{x^2 - 9}$ with $x = 3\sec\theta$

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5-03 Verify Trigonometric Identities

Verifying Trig Identities

- Show that trig identities are _____by _____one side into the other side

Guidelines

- Work with _____at a time. Start with the more _____side.
- Try _____, add _____, square a _____, etc.
- Use fundamental _____
- If the above doesn't work, try rewriting in _____and _____
- Try _____!

Verify $(1 + \sin\alpha)(1 - \sin\alpha) = \cos^2\alpha$

Verify $\sin^2\alpha - \sin^4\alpha = \cos^2\alpha - \cos^4\alpha$

Verify $\frac{\cot^2 t}{\csc t} = \csc t - \sin t$

Verify $\frac{1}{\sec x \tan x} = \csc x - \sin x$

Verify $\frac{\cos\theta \cot\theta}{1-\sin\theta} - 1 = \csc\theta$

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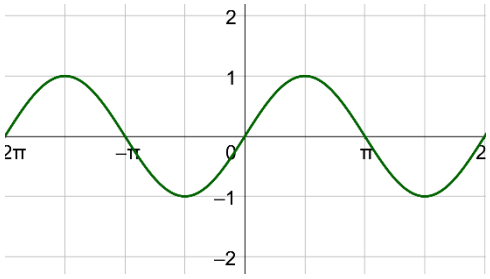
5-04 Solve Trigonometric Equations

- Main goal – Isolate a _____expression
  - Try _____to simplify
  - Try solving by _____



Number of solutions

- $\sin x = 0$
- Infinite solutions so describe
- _____

Solve $\sin x - \sqrt{2} = -\sin x$

Solve $4\sin^2 x - 3 = 0$

Solve $\sin^2 x = 2\sin x$

Solve $3\sec^2 x - 2\tan^2 x - 4 = 0$

Solve in the interval $[0, 2\pi)$: $\sin x + 1 = \cos x$

Solve on the interval $[0, 2\pi)$: $\sin 2x = \frac{\sqrt{3}}{2}$

Solve $4\tan^2 x + 5\tan x = 6$

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5-05 Sum and Difference Formulas

Sum and Difference Formulas

$\sin(u+v) = \sin u \cos v + \cos u \sin v$

$\cos(u+v) = \cos u \cos v - \sin u \sin v$

$\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$

$\sin(u-v) = \sin u \cos v - \cos u \sin v$

$\cos(u-v) = \cos u \cos v + \sin u \sin v$

$\tan(u-v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$

Use a sum or difference formula to find the exact value of $\tan 255°$

Find the exact value of $\cos 95° \cos 35° + \sin 95° \sin 35°$

Derive a reduction formula for $\sin\left(t + \frac{\pi}{2}\right)$

Find all solutions in $[0, 2\pi)$: $\cos\left(x - \frac{\pi}{3}\right) + \cos\left(x + \frac{\pi}{3}\right) = 1$

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5-06 Multiple Angle Formulas

Double-Angle Formulas

- $\sin 2u = 2 \sin u \cos u$
- $\cos 2u = \cos^2 u - \sin^2 u$
  $= 2\cos^2 u - 1$
  $= 1 - 2\sin^2 u$
- $\tan 2u = \frac{2\tan u}{1-\tan^2 u}$

If $\sin u = \frac{3}{5}$ and $0 < u < \frac{\pi}{2}$,

Find $\sin 2u$ $\cos 2u$

$\tan 2u$

Derive a triple angle formula for $\cos 3x$

Power-Reducing Formulas

- $\sin^2 u = \frac{1-\cos 2u}{2}$
- $\cos^2 u = \frac{1+\cos 2u}{2}$
- $\tan^2 u = \frac{1-\cos 2u}{1+\cos 2u}$

Rewrite $\cos^4 x$ as a sum of 1st powers of cosines.

Half-Angle Formulas

- $\sin\frac{u}{2} = \pm\sqrt{\frac{1-\cos u}{2}}$
- $\cos\frac{u}{2} = \pm\sqrt{\frac{1+\cos u}{2}}$
- $\tan\frac{u}{2} = \frac{1-\cos u}{\sin u}$
  $= \frac{\sin u}{1+\cos u}$

Find the exact value of $\cos 105°$

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5-07 Product-to-Sum Formulas

Product-to-Sum Formulas

- $\sin u \sin v = \frac{1}{2}(\cos(u - v) - \cos(u + v))$
- $\cos u \cos v = \frac{1}{2}(\cos(u - v) + \cos(u + v))$
- $\sin u \cos v = \frac{1}{2}(\sin(u + v) + \sin(u - v))$
- $\cos u \sin v = \frac{1}{2}(\sin(u + v) - \sin(u - v))$

Rewrite $\sin 5\theta \cos 3\theta$ as a sum or difference.

Sum-to-Product Formulas

- $\sin u + \sin v = 2 \sin\left(\frac{u+v}{2}\right) \cos\left(\frac{u-v}{2}\right)$
- $\sin u - \sin v = 2 \cos\left(\frac{u+v}{2}\right) \sin\left(\frac{u-v}{2}\right)$
- $\cos u + \cos v = 2 \cos\left(\frac{u+v}{2}\right) \cos\left(\frac{u-v}{2}\right)$
- $\cos u - \cos v = -2 \sin\left(\frac{u+v}{2}\right) \sin\left(\frac{u-v}{2}\right)$

Find the exact value of $\sin 195° + \sin 105°$

Solve on the interval $[0, 2\pi)$: $\sin 4x - \sin 2x = 0$

Verify $\frac{\sin 6x + \sin 4x}{\cos 6x + \cos 4x} = \tan 5x$